EXERCISE – III

SUBJECTIVE QUESTIONS

1. Show that the normals at the points (4a, 4a) & at the upper end of the latus rectum of the parabola $y^2 = 4ax$ intersect on the same parabola.

Sol.

2. Prove that the locus of the middle point of portion of a normal to $y^2 = 4ax$ intercepted between the curve & the axis is another parabola. Find the vertex & the latus rectum of the second parabola.

Sol.

3. Find the equations of the tangents to the parabola $y^2 = 16x$, which are parallel & perpendicular respectively to the line $2x - y + 5 = 0$. Find also the coordinates of their points of contact.

Sol.

4. A circle is described whose centre is the vertex and whose diameter is three-quarters of the latus rectum of a parabola $y^2 = 4ax$. Prove that the common chord of the circle and parabola bisects the distance between the vertex and the focus.

Sol.

5. Find the equations of the tangents of the parabola $y^2 = 12x$, which passes through the point (2, 5).

Sol.

6. Through the vertex O of a parabola $y^2 = 4x$, chords OP & OQ are drawn at right angles to one another. Show that for all positions of P, PQ cuts the axis of the parabola at a fixed point. Also find the locus of the middle point of PQ.

Sol.

7. Let S is the focus of the parabola $y^2 = 4ax$ and X the foot of the directrix, PP' is a double ordinate of the curve and PX meets the curve again in Q. Prove that P'Q passes through focus.

Sol.

8. Three normals to $y^2 = 4x$ pass through the point (15, 12). Show that if one of the normals is given by $y = x - 3$ & find the equations of the others.

Sol.

9. Find the equations of the chords of the parabola $y^2 = 4ax$ which pass through the point $(-6a, 0)$ and which subtends an angle of 45º at the vertex.

Sol.

10. Through the vertex O of the parabola $y^2 = 4ax$, a perpendicular is drawn to any tangent meeting it at P & the parabola at Q. Show that $OP \cdot OQ$ = constant.

Sol.

11. 'O' is the vertex of the parabola $y^2 = 4ax$ & L is the upper end of the latus rectum. If LH is drawn perpendicular to OL meeting OX in H, prove that the length of the double ordinate through H is $4a\sqrt{5}$.

Sol.

12. The normal at a point P to the parabola $y^2 = 4ax$ meets its axis at G. Q is another point on the parabola such that QG is perpendicular to the axis of the parabola. Prove that $QG^2 - PG^2$ = constant.

Sol.

13. If the normal at P(18, 12) to the parabola $y^2 = 8x$ cuts it again at Q, show that $9PQ = 80\sqrt{10}$

Sol.

14. Prove that, the normal to $y^2 = 12x$ at (3,6) meets the parabola again in (27, −18) & circle on this normal chord as diameter is $x^2 + y^2 - 30x + 12y - 27 = 0$.

Sol.

15. Find the equation of the circle which passes through the focus of the parabola $x^2 = 4y$ & touches it at the point (6, 9).

Sol.

16. P & Q are the points of contact of the tangents drawn from the point T to the parabola $y^2 = 4ax$. If PQ be the normal to the parabola at P, prove that TP is bisected by the directrix.

Sol.

17. From the point (–1, 2) tangent lines are drawn to the parabola $y^2 = 4x$. Find the equation of the chord of contact. Also find the area of the triangle formed by the chord of contact & the tangents.

Sol.

Read the information given and answer the question 18, 19, 20.

From the point P(h, k) three normals are drawn to the parabola $x^2 = 8y$ and m_1, m_2 and m_3 are the slopes of three normals

18. Find the algebraic sum of the slopes of these three normals.

Sol.

19. If two of the three normals are at right angles then the locus of point P is a conic, find the latus rectum of conic.

Sol.

20. If the two normals from P are such that they make complementary angles with the axis then the locus of point P is a conic, find a directrix of conic.

Sol.

21. Prove that the two parabolas $y^2 = 4ax$ & $y^2 = 4c(x - b)$ cannot have a common normal, other than the axis, unless $\frac{b}{(a-c)} > 2$

Sol.

22. Find the condition on 'a' & 'b' so that the two tangents drawn to the parabola $y^2 = 4ax$ from a point are normals to the parabola $x^2 = 4by$.

Sol.